Numeric Response Questions

Trigonometric Equations

Q.1 If $5\cos 2\theta + 2\cos^2 \frac{\theta}{2} + 1 = 0, -\pi < \theta < \pi$ then find number of solutions.

Q.2 Find the number of solutions of the equation $\sin^4 x = 1 + \tan^4 x$ in $(0,4\pi)$.

Q.3 Find the number of solutions of the equation $\sin^2 x + 3\sin x + 2 = 0$ in the interval $(-\pi, \pi)$.

Q.4 Find the total number of solutions of $16^{\sin^1 x} + 16^{\cos^2 x} = 10$ in $x \in [0,3\pi]$.

Q.5 If $\sin^2 x - \cos x = 1/4$, if the sum of values of x between 0 and 2π is $k\pi$ then find k.

Q.6 If exp $[(\sin^2 x + \sin^4 x + \sin^6 x + \cdots.,)$ In 2] satisfies the equation $y^2 - 9y + 8 = 0$, and the value of $\frac{\cos x}{\cos x + \sin x}, 0 < x < \frac{\pi}{2}$, is $\frac{a-1}{2}$ then find a

Q.7 Find the number of integral values of k for which the equation $7\cos x + 5\sin x = 2k + 1$ has a solution.

Q.8 Find the total number of solutions to $|\cot x| = \cot x + \frac{1}{\sin x}, x \in [0,3\pi]$.

Q.9 If $\sin \theta = \frac{1}{2}$ and $\cos \theta = -\frac{\sqrt{3}}{2}$ and the general value of θ is $2n\pi + \frac{k\pi}{\lambda} (n \in Z)$ then find $k + \lambda$

Q.10 If $\sin^2 x - \cos^2 x\sin x + 2\sin^2 x + \sin x = 0, \forall x \in [0,3\pi$ | then find number of solutions.

Q.11 If the most general solution of $2^{1+\cos x|+6a^2x+|\cos x|^3+\cdots x} = 4$, is $n\pi \pm \frac{\pi}{k}$ then find k.

Q.12 The values of x between 0 and 2π which satisfy the equation $\sin x\sqrt{8\cos^2 x} = 1$ are in AP. If the common difference of the AP is $\frac{\pi}{k}$ then find k.

Q.13 Find the total number of aolutions to $\tan x + \cot x = 2\text{cosec } x$ in $[-2\pi, 2\pi]$.

Q.14 If general solution of $\cot \theta - \tan \theta = \sec \theta$ is $n\pi + (-1)n\frac{\pi}{k}, n \in Z$ then find k.

Q.15 If general solution of $7\cos^2 \theta + 3\sin^2 \theta = 4$ is $n\pi \pm \frac{\pi}{k}, n \in Z$ then find k.

ANSWER KEY

1. 4.00 **2.** 0.00 **3.** 1.00 **4.** 12.00 **5.** 2.00 **6.** 1.73 **7.** 8.00

8. 2.00 **9.** 11.00 **10.** 4.00 **11.** 3.00 **12.** 4.00 **13.** 4.00 **14.** 6.00

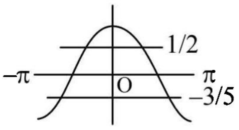
15. 3.00

Hints & Solutions

1. $5(2\cos^2\theta - 1) + 1 + \cos\theta + 1 = 0$

$\Rightarrow 10\cos^2\theta + \cos\theta - 3 = 0$

$\Rightarrow \left(\cos\theta + \frac{3}{5}\right)\left(\cos\theta - \frac{1}{2}\right) = 0$



2. $0 \le \sin^4 x \le 1$

$1 \le 1 + \tan^4 x < \infty$

LHS = RHS = 1

$\sin^4 x = 1$ and $1 + \tan^4 x = 1$

$\sin^2 x = 1$ and $\tan x = 0$

which is not possible

3. $\sin^2 x + 3\sin x + 2 = 0$

$\Rightarrow (\sin x + 1)(\sin x + 2) = 0$

$\Rightarrow \sin x = -1$, As $\sin x \ne -2$

$x = -\frac{\pi}{2}$

4. If $16^{\sin^2 x} + \frac{16}{16^{\sin^2 x}} = 10$

let $16^{\sin^2 x} = y$

$\because\ y + \frac{16}{y} = 10$

$y^2 - 10y + 16 = 0$

$y = 8$ and $y = 2$

$16^{\sin^2 x} = 8,\ 16^{\sin^2 x} = 2$

$\therefore \sin^2 x = \frac{3}{4},\ \sin^2 x = \frac{1}{4}$

There are 12 solutions in $[0, 3\pi]$

5. $\text{Sin}^2 x - \cos x = \frac{1}{4}$

$\Rightarrow 1 - \cos^2 x - \cos x = \frac{1}{4}$

$\Rightarrow 4\cos^{+2} x + y\cos x - 3 = 0$

$\Rightarrow (2\cos x + 3)(2\cos x - 1) = 0$

$\Rightarrow 2\cos x = 1\ (\because\ \cos x \ne -3/2)$

$\Rightarrow \cos x = 1/2$

$\Rightarrow \cos x = \cos \pi/3 \Rightarrow x = 2n\pi \frac{\pm\pi}{3}, n \in I$

$\therefore x = \frac{\pi}{3}, \frac{5\pi}{3} \in [0, 2\pi]$

6. $\sin^2 x + \sin^4 x + \sin^6 x + \ldots\ldots \infty$

$= \frac{\sin^2 x}{1 - \sin^2 x} = \tan^2 x$

$\Rightarrow \exp[(\sin^2 x + \sin^4 x + \ldots\ldots)\ln 2]$

$= e^{\tan^2 x \ln 2}$

The given equation is $y^2 - 9y + 8$

$\Rightarrow (y - 1)(y - 8) = 0$

Either $y = 1 \Rightarrow 2^{\tan^2 x} = 1 = 2^0$

$\Rightarrow \tan^2 x = 0$, but $x \in \left(0, \frac{\pi}{2}\right)$, $\therefore$ neglecting

$x = 0$

Or $y = 2^3 \Rightarrow \tan^2 x = 3 \Rightarrow \tan x = \pm\sqrt{3}$

$\Rightarrow x = \frac{\pi}{3}$

as, $0 < x < \frac{\pi}{2}$

$\Rightarrow \frac{\cos x}{\cos x + \sin x} = \frac{1/2}{1/2 + \sqrt{3}/2}$

$= \frac{1}{\sqrt{3} + 1} = \frac{\sqrt{3} - 1}{2}$.

7. $7\cos x + 5\sin x = 2k + 1$

equation has a solution only when

$-\sqrt{49+25} \le 2k + 1 \le \sqrt{49+25}$

$-\sqrt{74} \le 2k + 1 \le \sqrt{74}$

$-8.5 \le 2k + 1 \le 8.5$

$-4.75 \le k \le 3.75$

$k = -4, -3, -2, -1, 0, 1, 2, 3$

Thus there are 8 values.

8. $|\cot x| = \cot x + \frac{1}{\sin x}$

if $\cot x > 0 \Rightarrow \cot x = \cot x + \frac{1}{\sin x} = 0$

$\Rightarrow \frac{1}{\sin x} = 0$ which is not possible

if $\cot x \le 0 \Rightarrow -\cot x = \cot x + \frac{1}{\sin x}$

$\Rightarrow -2\cot x = \frac{1}{\sin x}$

$\Rightarrow \cos x = -\frac{1}{2} \Rightarrow x = \frac{2\pi}{3}, \frac{8\pi}{3}$

10. $\sin^4 x + \sin^3 x + 2\sin^2 x = 0$

$\sin^2 x(\sin^2 x + \sin x + 2) = 0$

$x = n\pi$

11. $2^{\frac{1}{1-|\cos x|}} = 2^2$

$\Rightarrow \frac{1}{1-|\cos x|} = 2$

$\Rightarrow 2 - 2|\cos x| = 1$

$\Rightarrow |\cos x| = \frac{1}{2}$

$x = n\pi \pm \frac{\pi}{3}$

12. $2\sin x\,|\cos x| = \frac{1}{\sqrt{2}}$

if $\cos x > 0$ then $\sin 2x = \frac{1}{\sqrt{2}}$

$\Rightarrow x = \frac{\pi}{8}, \frac{3\pi}{8}$

if $\cos x < 0$, then $\sin 2x = -\frac{1}{\sqrt{2}}$

$\Rightarrow x = \frac{5\pi}{8}, \frac{7\pi}{8}$

so, $x = \frac{\pi}{8}, \frac{3\pi}{8}, \frac{5\pi}{8}, \frac{7\pi}{8}$

13. $\tan x + \cot x = 2\operatorname{cosec} x$

$\Rightarrow \frac{\sin x}{\cos x} + \frac{\cos x}{\sin x} = \frac{2}{\sin x}$

$\Rightarrow \frac{1}{\sin x\cos x} = \frac{2}{\sin x}$

$\Rightarrow \cos x = \frac{1}{2}$

$\Rightarrow x = \pm\frac{\pi}{3}, \pm\frac{5\pi}{3}$

thus, there are four solutions

14. $\cos^2\theta - \sin^2\theta = \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$\sin\theta = \frac{1}{2}$, $\sin\theta = -1$ (rejected)

$\theta = n\pi + (-1)^n \frac{\pi}{6}$

15. $4\cos^2\theta = 1$

$\theta = n\pi \pm \frac{\pi}{3}$